Multivariable Calculus

Quiz 8 SOLUTIONS

We want to find the absolute maximum and minimum values of

$$f(x, y) = x^2 + y^2 - 2x + 2y + 1$$

on the region $x^2 + y^2 \leq 8$. Follow the steps below to find these values.

1) Find all critical points of f. Which of these critical points is in the region (i.e. satisfy $x^2 + y^2 \leq 8$)?

   Solution: We need to solve

$$\boldsymbol{\nabla} f(x, y) = \langle 2x - 2,\ 2y + 2\rangle = \langle 0, 0\rangle.$$

   The first component gives $2x - 2 = 0 \longleftrightarrow x = 1$ while the second component gives $2y+2 = 0 \longleftrightarrow y = -1$. So, the only critical point is $(1, -1)$. Since $(1)^2+(-1)^2 = 2 \leq 8$, this critical point is inside the region.

2) Use Lagrange Multipliers to identify critical points on the boundary

$$g(x, y) = x^2 + y^2 = 8.$$

   Solution: We have the equations $\boldsymbol{\nabla} f(x, y) = \lambda \boldsymbol{\nabla} g(x, y)$ along with the equation of constraint. That gives us three equations in total.

$$\begin{aligned} 2x - 2 &= 2\lambda x \\ 2y + 2 &= 2\lambda y \\ x^2 + y^2 &= 8 \end{aligned} \qquad \longrightarrow \qquad \begin{aligned} (1-\lambda)x &= 1 \\ (1-\lambda)y &= -1 \\ x^2 + y^2 &= 8 \end{aligned}$$

   Notice that λ cannot equal 1 (as we would get $0 = 1$). The first two equations then imply $y = -x$. Substituting this into the constraint tells us that $x = \pm 2$. That means we have 2 critical points on the boundary: $(2, -2)$ and $(-2, 2)$.

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3) Fill out the chart below to identify the maximum and minimum values of f over the region. Check Absolute Max or Absolute Min for the appropriate rows. (NOTE: You should have found at least 3 candidate points.)

	(x, y)	$f(x, y)$	Abs. Max?	Abs. Min?
1	$(1, -1)$	-1		✓
2	$(2, -2)$	1		
3	$(-2, 2)$	17	✓	